Steady State Analysis of PID Controlled Boost Converter using State Space Averaging Technique

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Abstract — This paper discusses steady state analysis of Boost Converter using State space averaging technique and small signal analysis. Proportional Integral Derivative controller is designed to simplify the compensation of the DC-DC converter. The converter is modeled using state space averaging technique. Transfer function obtained from small signal analysis is used to plot root locus and improve steady state response by choosing a proper value of gain K and make the system stable.

Keywords — SSA (state space averaging), PID (Proportional Integral Derivative)

Introduction

DC-DC power converters are employed in a variety of applications, including power supplies for personal computers, office equipment, spacecraft power systems, laptop computers, and telecommunications equipment, as well as dc motor drives, PV arrays[1]. The input to a dc-dc converter is an unregulated dc voltage. High efficiency is invariably required, since cooling of inefficient power converters is difficult and expensive. This is achieved using switched-mode chopper circuits whose elements dissipate negligible power. Pulse-width modulation (PWM) allows control and regulation of the total output voltage. Modeling and analysis of switching DC-DC converters can be either numerical or analytical. In numerical techniques, various algorithms or circuit simulators are used to produce quantitative results. These methods...
are easy to use. Analytic techniques, on the other hand, provide analytic expressions representing the operation and performance of the converters. Perhaps the most popular continuous-time technique is the small-signal analysis, which uses either State-space averaging or PWM switch modeling. The PID control method can be implemented by using digital or analog methods to control parameters of converter such as voltage, current, etc. In analog control, analog components such as resistors, capacitors, inductors and operational-amplifiers is used for implementation of controlling algorithm, while in digital control, microprocessors are used and controlling algorithm can be programmed and feed to microprocessor. Digital control system offer many advantages over their Analog counterparts.

CONCEPTUAL BACKGROUND

Basic Circuit of the Boost Converter

The boost converter is shown in Fig.1. This is another switching converter that operates by periodically opening and closing an electronic switch. It is called a boost converter because the output voltage is larger than the input. The switching period is T, and the switch is closed for time DT and open for (1-D) T. The inductor current is continuous (always positive). The capacitor is very large, and the output voltage is held constant at voltage Vo. The analysis proceeds by examining the inductor voltage and current for the switch closed and again for the switch open.

![Fig.1.Circuit diagram of Boost Converter](image)

Need for closed loop control of the boost Converter

Closed-loop systems have many advantages over open-loop systems. The primary advantage of a closed-loop feedback control system is its ability to reduce a system’s sensitivity to external disturbances. Closed-loop systems are designed to automatically achieve and maintain the desired output condition by comparing it with the actual condition. It does this by generating an error signal which is the difference between the output and the reference input. In other words, a “closed-loop system” is a fully automatic control system in which its control action being dependent on the output in some way. Thus by forming a closed loop we can
reduce errors by automatically adjusting the systems input, improve stability of an unstable system and to produce a reliable and repeatable performance.

State Space Analysis

Large Signal Modeling of Boost Converter

Two modes of Boost converter are

(i) Mode 1: When switch is closed and diode is reverse biased

(ii) Mode 2: When switch is open and diode is forward biased

The following table describes the analysis of these modes.

Table 1. Equations for different modes of Boost Converter

<table>
<thead>
<tr>
<th>Mode 1</th>
<th>Mode 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{di}{dt} = \frac{V_s}{L} )</td>
<td>( \frac{di}{dt} = \frac{1}{L} (V_s - V_o) )</td>
</tr>
<tr>
<td>( \frac{dV_c}{dt} = -\frac{V_c}{RC} )</td>
<td>( \frac{dV_c}{dt} = \frac{i}{C} - \frac{V_c}{RC} )</td>
</tr>
</tbody>
</table>

Mode 1: When switch is closed and diode is reverse biased.

\[ \dot{X} = [A_1]X + [B_1]U \] \hspace{1cm} (1)

State space matrix for Mode 1 of Boost Converter

\[
\begin{bmatrix}
  i_l \\
  V_c
\end{bmatrix} =
\begin{bmatrix}
  0 & 0 \\
  0 & -1/RC
\end{bmatrix}
\begin{bmatrix}
  i \\
  V_c
\end{bmatrix} +
\begin{bmatrix}
  1/L \\
  0
\end{bmatrix}
\begin{bmatrix}
  V_s
\end{bmatrix}
\]
Mode 2: When switch is open, the diode is forward biased.

![Equivalent circuit for switch open](image)

Steady state equation for mode2 is given by Equ. (2)

$$\dot{X} = [A_2]X + [B_2]U$$  \hspace{1cm} (2)

State space matrix for Mode 2 of Boost Converter

$$\begin{bmatrix} \dot{i} \\ \dot{V_c} \end{bmatrix} = \begin{bmatrix} 0 & -1/L \\ 1/C & -1/RC \end{bmatrix} \begin{bmatrix} i \\ V_c \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} [V_s]$$

Matrices $[A]$ and $[B]$ are obtained by using (3) and (4) respectively using state space averaging technique

$$A = A_1D + A_2(1-D)$$ \hspace{1cm} (3)

$$B = B_1D + B_2(1-D)$$ \hspace{1cm} (4)

Steady state equation for boost converter is given by

$$\dot{X} = [A]X + [B]U$$  \hspace{1cm} (5)

State space matrix for Boost Converter

$$\begin{bmatrix} \dot{i} \\ \dot{V_c} \end{bmatrix} = \begin{bmatrix} 0 & -1/L(1-D) \\ (1-D)/C & -1/RC \end{bmatrix} \begin{bmatrix} i \\ V_c \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} [V_s]$$

**B. Small signal modeling of Boost Converter**

Small signal and steady state analysis of the system[3] are perturbed by assuming the variables are perturbed around steady state operating point.
\[ x = x + \hat{x} \]
\[ d = D + \hat{d} \]
\[ u = U + \hat{u} \]

\( X, D, U \) are steady state values. \( x, d, u \) Are small signal values

For steady state response

\[ \dot{x} = 0 \]

And small signal values are zero \( \dot{x} = 0 = AX + Bu \)

Thus we obtain the input and output parameters by (6) and (7) respectively.

\[ X = [A]^{-1} [B] [u] \]  
\[ y = -[C][A]^{-1} [B] [u] \]  

Small signal analysis starts by recognizing that derivative of steady state component is zero and we obtain Equ. (8)

\[ \dot{x} = \dot{X} + \hat{x} = 0 + \hat{x} = \hat{x} \]  

Substituting steady state and small signal equations we get (9)

\[ \begin{bmatrix} \hat{i} \\ \hat{V}_c \end{bmatrix} = \begin{bmatrix} 0 & \frac{-i(1-D)}{L} \\ (1-D) & \frac{-1}{RC} \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{V}_c \end{bmatrix} + \begin{bmatrix} \frac{1}{L} & \frac{V_c}{L} \\ \frac{-i}{C} & 0 \end{bmatrix} \begin{bmatrix} \hat{V}_g \\ \hat{d} \end{bmatrix} \]

Thus Capacitor voltage \( V_c \) and inductor current \( i \) are given by Equ. (10) and Equ.(11) respectively

\[ V_c = V_g / (1 - D) \]  

\[ i = \frac{V_c}{L} \]
$i_r = \frac{V_g}{R(1 - D)^2}$ \hspace{2cm} (11)

**Circuit Parameters Used For Simulation**

Specifications of various parameters taken from [4], used for simulation and in program are given below

- Source Voltage ($V_s$) = 12V
- Frequency ($F$) = 25KHz
- Resistance ($R$) = 50ohms
- Capacitance ($C$) = 48μF
- Inductance ($L$) = 120μH

**Simulink Open loop model of Boost Converter**

Implementation of Boost Converter is done in SIMULINK using the equations obtained from Large signal modeling. Simulink subsystem model is shown in Fig.4 and complete system in Fig.5. Outputs i.e. inductor current ($i_l$) and capacitor voltage are obtained. Results are shown in fig.6 and fig.7. Steady state value is reached at 0.25sec.

![Simulink model of Boost subsystem](image)
Simulink Closed loop model of Boost Converter

Block diagram of closed loop boost converter is shown in Fig.6. Output voltage is fed back to the controller through Feedback system and compared with reference voltage. PID controller generates the desired value of duty ratio so as to obtain the desired voltage. Simulink model of Closed loop converter is shown in Fig.7.

Fig.5 Simulink model of Open Loop Boost Converter

Fig.6 Block diagram of closed loop Boost converter

Fig.7 Simulink model of Closed Loop Boost Converter
Results And Discussion

Open loop results of Boost Converter

Comparison of theoretical and obtained values is done in Table 2. Thus verifying the results. Fig.8 and 9 gives the output inductor current and capacitor voltage respectively.

<table>
<thead>
<tr>
<th></th>
<th>Theoretical values</th>
<th>Obtained values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{\text{max}}$</td>
<td>2.7A</td>
<td>2.5A</td>
</tr>
<tr>
<td>$I_{\text{min}}$</td>
<td>0.3A</td>
<td>0.3A</td>
</tr>
<tr>
<td>$V_0$</td>
<td>25A</td>
<td>24A</td>
</tr>
</tbody>
</table>

Table2. Results of Boost Converter

Fig.8 Current v/s time for Boost Converter

Fig.9 Transient response of open loop Boost Converter
Root Locus Plot

Root Locus for transfer function of the system obtained is plotted as shown in Fig.10. Particular gain value K is chosen from the root locus. Step response for the closed loop system is plotted so as to check the stability of the system. By choosing appropriate gain value K steady state is reached at a faster rate.

Transfer function obtained from program executed is given by Equ.(12)

\[
G_p(s) = \frac{-31250(s - 6.667e004)}{s(s^2 + 416.7s + 2.778e007)}
\]  
(12)

Controller transfer function is given as below by Equ. (13)

\[
G_c(s) = \frac{1}{s}
\]  
(13)

![Root locus plot](image)

*Fig.10 Root locus for transfer function of closed loop Boost converter*

Transfer function of the system

\[
G_p(s) * G_c(s) = \frac{-25199.552(s - 6.667e004)}{(s + 60.58)(s^2 + 356s + 2.773e007)}
\]  
(14)

Closed loop results of Boost Converter

For the particular value of gain step response is plotted so as to reach the desired voltage at a faster rate.
From Fig.11 we observe that steady state value is reached at 0.5 sec. Thus improving transient response of the system.

Fig. 11. Step response of closed loop Boost Converter

Fig.12 and 13 give the plot of Capacitor voltage and Inductor current of closed loop Boost converter.

Fig.12 Voltage V/s time of closed loop converter

Fig.13 Current V/s time of Closed Loop Converter

Fig.12 and 13 give the plot of Capacitor voltage and Inductor current of closed loop Boost converter.

V. CONCLUSION and FUTURE SCOPE

All of the specifications stated previously have been met by the Boost Converter design. Modeling of open loop and closed loop Boost converter simulations using calculated parameters were performed and corresponding waveforms are obtained. It is found that by adding PID controller transient response is improved and steady state value is reached at a faster rate i.e. from 0.25 sec to 0.005 sec. Hence Justifying the use of controllers in a system to meet the desired specifications.

Modeling of closed loop Boost Converter for practical conditions can be carried out. Using this Boost converter for higher voltage applications and applying this for photovoltaic array for MPPT tracking.
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